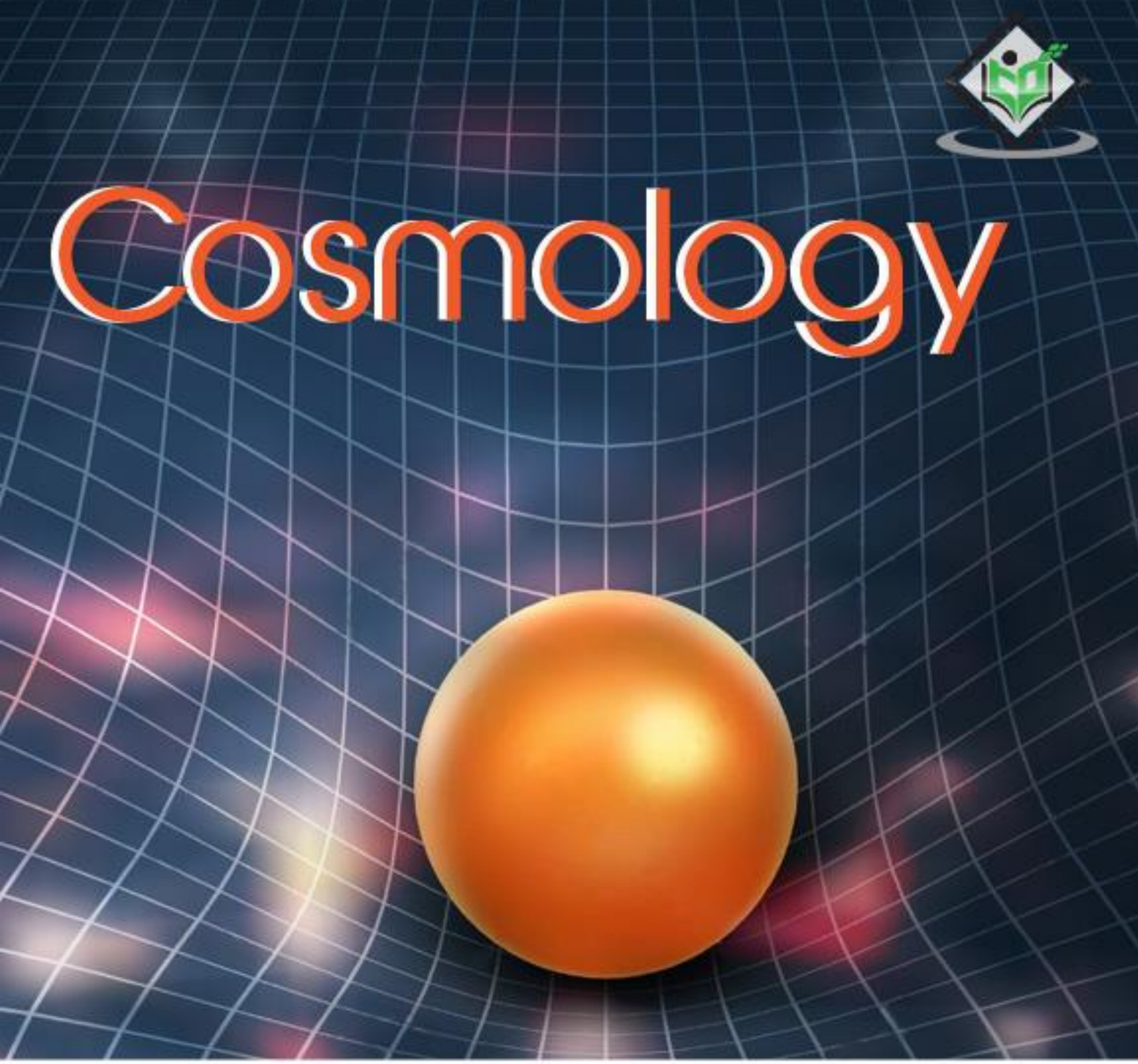
Cosmology



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About the Tutorial

Cosmology is the science and study of the origin, current state and the future of the Universe. This field has been revolutionized by many discoveries made during the past century.

This tutorial will attempt at explaining the basic cosmology and summarize the discoveries made in this regard. It gives a deep insight into the Big Bang Theory and the attempts made to discover the Extrasolar Planets. The mysterious CMB (Cosmic Microwave Background Radiation) and its anisotropies which have intrigued astronomers for several years has been explained in detail here. This tutorial will be very useful for budding scientists and astronomers.

Audience

This tutorial is aimed at students and readers who want to be able to understand and appreciate the Cosmological Phenomena we observe in the night sky. It will be of great importance to young astronomers, who wish to explore the night sky for finding astronomical objects.

Prerequisites

This tutorial requires prior understanding of Basic Astronomy and the Relativity Theory. It will also require you to apply concepts of Basic Physics, Integral and Differential Calculus, and Differential Equations.

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1. Cosmology – The Expanding Universe

Cosmology is the study of the universe. Tracing back in the time, there were several school of thoughts regarding the origin of the universe. Many scholars believed in the **Steady State Theory**. As per this theory, the universe was always the same, it had no beginning.

While there were a group of people who had faith in the **Big Bang Theory**. This theory predicts the beginning of the universe. There were evidences of hot left out radiation from the Big Bang, which again supports the model. The Big Bang Theory predicts the abundance of light elements in the universe. Thus, following the famous model of Big Bang, we can state that the universe had a beginning. We are living in an expanding universe.

The Hubble Redshift

In the early 1900's, the state of the art telescope, **Mt Wilson**, a 100-inch telescope, was the biggest telescope then. Hubble was one of the prominent scientists, who worked with that telescope. He discovered there were galaxies outside the Milky Way. **Extragalactic Astronomy** is only 100 years old. Mt Wilson was the biggest telescope until Palmer Observatory was built which had a 200-inch telescope.

Hubble was not the only person observing galaxies outside the Milky Way, Humason helped him. They set out on measuring the spectra of nearby galaxies. They then observed a galactic spectrum was in the visible wavelength range with continuous emission. There were emission and absorption lines on top of the continuum. From these lines, we can make an estimate if the galaxy is moving away from us or towards us.

When we get a spectrum, we assume the strongest line is coming from **H-α**. From literature, the strongest line should occur at **6563 Å**, but if the line occurs somewhere around **7000Å**, we can easily say it is redshifted.

From the **Special Theory of Relativity**,

$$1 + z = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

where, Z is the redshift, a dimensionless number and v is the recession velocity.

$$\frac{\lambda_{obs}}{\lambda_{rest}} = 1 + z$$

Hubble and Humason listed down **22 Galaxies** in their paper. Nearly all these galaxies exhibited redshift. They plotted the velocity (km/s) vs distance (Mpc). They observed a linear trend and Hubble put forward his famous law as follows.

$$v_r = H_o d$$

This is the **Hubble Redshift Distance Relationship**. The subscript **r** indicates expansion is in the radial direction. While, vr is the receding velocity, Ho is the Hubble parameter, **d** is the distance of the galaxy from us. They concluded far away galaxies recede faster from us, if the rate of expansion for the universe is uniform.

The Expansion

Everything is moving away from us. The galaxies are not stationary, there is some expansion harmonic always. The units of the Hubble parameter are $km\ sec^{-1}Mpc^{-1}$. If one goes out a distance of – 1 Mpc, galaxies would be moving at the rate of 200 kms/sec. The Hubble parameter gives us the rate of expansion. As per Hubble and Humason, the value of Ho is 200 kms/sec/Mpc.

The data showed all galaxies are moving away from us. Thus, it is apparent that we are at the center of the universe. But Hubble didn't make this mistake, as per him, in whichever galaxy we live, we would find all other galaxies moving away from us. Thus, the conclusion is that the space between galaxies expand and there is no center of the universe.

The expansion is happening everywhere. However, there are some forces that are opposing expansion. Chemical bonds, gravitational force and other attractive forces are holding objects together. Earlier all the objects were close together. Considering the Big Bang as an impulsive force, these objects are set to move away from each other.

Time Scale

At local scales, Kinematics is governed by Gravity. In the original Hubble's law, there were some galaxies which showed blue-shift. This can be credited to combined gravitational potential of the galaxies. Gravity has decoupled things from the Hubble's law. The Andromeda Galaxy is coming towards to us. Gravity is trying to slow things down. Initially the expansion was slowing down, now it is speeding up.

There was a **Cosmic Jerk** because of this. Several estimates to the Hubble parameter has been made. It has evolved over the 90 years from 500 kms/sec/Mpc to 69 kms/sec/Mpc. The disparity in the value was because of the underestimation of distance. The **Cepheid Stars** were used as distance calibrators, however there are different types of Cepheid stars and this fact was not considered for the estimation of the Hubble parameter.

Hubble Time

The Hubble constant gives us a realistic estimate of the age of the universe. The Ho would give the age of the universe provided the galaxies have been moving with the same velocity. The inverse of Ho gives us Hubble time.

$$t_H = \frac{1}{H_o}$$

Replacing the present value of Ho, t_H**=14** billion years. Rate of expansion has been constant throughout the beginning of the Universe. Even if this is not true, Ho gives a useful limit on the age of the universe. Assuming a constant rate of expansion, when we plot a graph between distance and time, the slope of the graph is given by velocity.

In this case, the Hubble time is equal to the actual time. However, if the universe had been expanding faster in the past and slower in the present, the Hubble time gives an upper limit of age of the universe. If the universe was expanding slowly before, and speeding up now, then the Hubble time will give a lower limit on age of the universe.

- $t_H = t_{age}$ – if rate of expansion is constant.
- $t_H > t_{age}$ – if universe has expanded faster in the past and slower in the present.
- $t_H < t_{age}$ – if universe has expanded slower in the past and faster in the present.

Consider a group of 10 galaxies which are at 200 Mpc from another group of galaxies. The galaxies within a cluster never conclude that the universe is expanding because kinematics within a local group is governed by gravitation.

Points to Remember

- Cosmology is the study of the past, present and future of our Universe.
- Our universe is ~14 billion years old.
- The universe is continuously expanding.
- Hubble parameter is a measure of the age of the universe.
- Current value of $\mathbf{H_o}$ is 69 kms/sec/Mpc.

2. Cosmology – Cepheid Variables

For a very long time, nobody considered galaxies to be present outside our Milky Way. In 1924, Edwin Hubble detected **Cepheid's** in the Andromeda Nebula and estimated their distance. He concluded that these "Spiral Nebulae" were in fact other galaxies and not a part of our Milky Way. Hence, he established that M31 (Andromeda Galaxy) is an Island Universe. This was the birth of **Extragalactic Astronomy**.

Cepheid's show a **periodic dip in their brightness**. Observations show that the period between successive dips called the period of pulsations is related to luminosity. So, they can be used as distance indicators. The main sequence stars like the Sun are in Hydrostatic Equilibrium and they burn hydrogen in their core. After hydrogen is fully burned, the stars move towards the Red Giant phase and try to regain their equilibrium.

Cepheid Stars are post Main Sequence stars that are transiting from the Main Sequence stars to the Red Giants.

Classification of Cepheids

There are 3 broad classes of these pulsating variable stars:

- **Type-I Cepheids** (or Classical Cepheids): period of 30-100 days.
- **Type-II Cepheids** (or W Virginis Stars): period of 1-50 days.
- **RR Lyrae Stars**: period of 0.1-1 day.

At that time, Hubble was not aware of this classification of variable stars. That is why there was an overestimation of the Hubble constant, because of which he estimated a lower age of our universe. So, the recession velocity was also overestimated. In Cepheid's, the disturbances propagate radially outward from the centre of the star till the new equilibrium is attained.

Relation between Brightness and Pulsation Period

Let us now try to understand the physical basis of the fact that higher pulsation period implies more brightness. Consider a star of luminosity L and mass M.

We know that –

$$L \propto M^{\alpha}$$

where α = 3 to 4 for low mass stars.

From the **Stefan Boltzmann Law**, we know that –

$$L \propto R^2 T^4$$

If **R** is the radius and **c_s** is the speed of sound, then the period of pulsation **P** can be written as –

$$P = R / c_s$$

But the speed of sound through any medium can be expressed in terms of temperature as –

$$c_s = \sqrt{\frac{\gamma P}{\rho}}$$

Here, **γ** is 1 for isothermal cases.

For an ideal gas, P=nkT, where k is the **Boltzmann Constant**. So, we can write –

$$P = \frac{\rho k T}{m}$$

where **ρ** is the density and **m** is the mass of a proton.

Therefore, period is given by –

$$P \cong \frac{R m^{\frac{1}{2}}}{(kT)^{\frac{1}{2}}}$$

Virial Theorem states that for a stable, self-gravitating, spherical distribution of equal mass objects (like stars, galaxies), the total kinetic energy **k** of the object is equal to minus half the total gravitational potential energy **u**, i.e.,

$$u = -2k$$

Let us assume that virial theorem holds true for these variable stars. If we consider a proton right on the surface of the star, then from the virial theorem we can say –

$$\frac{GMm}{R} = mv^2$$

From Maxwell distribution,

$$v = \sqrt{\frac{3kT}{2}}$$

Therefore, period –

$$P \sim \frac{RR^{\frac{1}{2}}}{(GM)^{\frac{1}{2}}}$$

which implies

$$P \propto \frac{R^{\frac{3}{2}}}{M^{\frac{1}{2}}}$$

We know that – $M \propto L^{1/\alpha}$

Also, $R \propto L^{1/2}$

So, for **β>0**, we finally get – $P \propto L^{\beta}$

Points to Remember

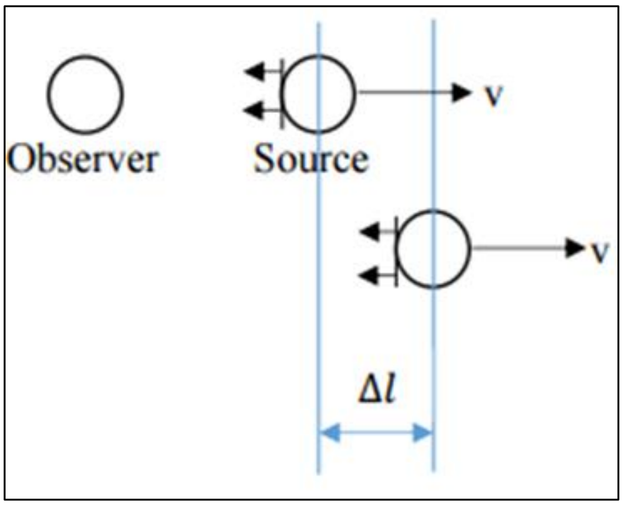
- Cepheid Stars are post Main Sequence stars that are transiting from the Main Sequence stars to Red Giants.
  -
- Cepheid's are of 3 types: Type-I, Type-II, RR-Lyrae in decreasing order of pulsating period.
  -
- Pulsating period of Cepheid is directly proportional to its brightness (luminosity).

3. Cosmology – Redshift and Recessional Velocity

Hubble's observations made use of the fact that radial velocity is related to shifting of the **Spectral Lines**. Here, we will observe four cases and find a relationship between Recessional Velocity (v_r) and Red Shift (z).

Case 1: Non-Relativistic Case of Source moving

In this case, *v* is much less than *c*. Source is emitting some signal (sound, light, etc.), which is propagating as **Wavefronts**. The time interval between the sending of two consecutive signals in the source frame is **Δts**. The time interval between the reception of two consecutive signals in the observer frame is **Δto**.



If both the observer and source are stationary, then Δts = Δto, but this is not the case here. Instead, the relation is as follows.

$$\Delta t_o = \Delta t_s + \frac{\Delta l}{c}$$

Now, $\boldsymbol{\Delta l = v\Delta ts}$

Also, since (wave speed x time) = wavelength, we get

$$\frac{\Delta t_o}{\Delta t_s} = \frac{\lambda_o}{\lambda_s}$$

From the above equations, we get the following relation –

$$\frac{\lambda_o}{\lambda_s} = 1 + \frac{v}{c}$$

where λ_s is the wavelength of the signal at the source and λ_o is the wavelength of the signal as interpreted by the observer.

Here, since the source is moving away from the observer, **v** is positive.

Red shift –

$$z = \frac{\lambda_o - \lambda_s}{\lambda_s} = \frac{\lambda_o}{\lambda_s} - 1$$

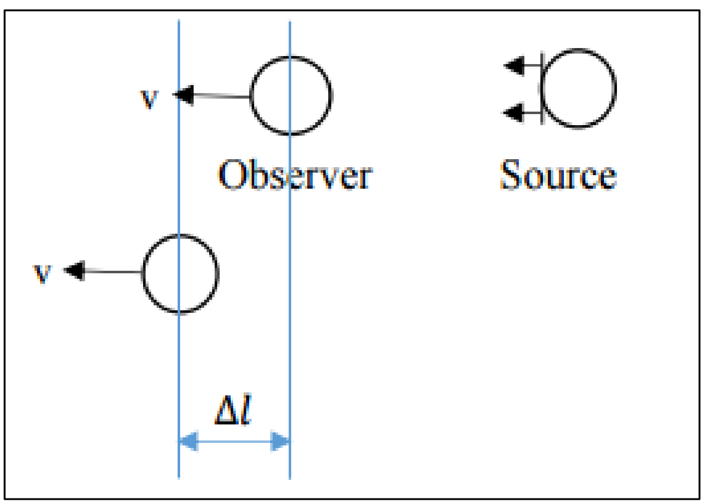
From the above equations, we get Red shift as follows.

$$\boxed{z = \frac{v}{c}}$$

Case 2: Non-Relativistic Case of Observer Moving

In this case, *v* is much less than *c*. Here, Δl is different.

$$\Delta l = v\, \Delta t_o$$



On simplification, we get –

$$\frac{\Delta t_o}{\Delta t_s} = \left(1 - \frac{v}{c}\right)^{-1}$$

We get Red shift as follows –

$$z = \frac{v/c}{1 - v/c}$$

Since **v << c**, the red shift expression for both Case I and Case II are approximately the same.

Let us see how the red shifts obtained in the above two cases differ.

$$z_{II} - z_I = \frac{v}{c}\left[\frac{1}{1 - v/c} - 1\right]$$

Hence, $zII - zI$ is a very small number due to the $(v/c)^2$ factor.

This implies that, if v << c, we cannot tell whether the source is moving, or the observer is moving.

Let us now understand the **Basics of STR** (Special Theory of Relativity):

- Speed of light is a constant.
  -
- When the source (or observer) is moving with a speed comparable to the speed of light, relativistic effects are observed.
  -
- Time dilation: $\Delta t_o = \gamma \Delta t_s$
  -
- Length contraction: $\Delta l_o = \Delta t_s / \gamma$
  -
- Here, γ is the **Lorrentz factor**, greater than 1.

$$\gamma = \frac{1}{\sqrt{1 - (v^2/c^2)}}$$

Case 3: Relativistic Case of Source Moving

In this case, *v* is comparable to *c*. Refer to the same figure as in Case I. Due to the relativistic effect, time dilation is observed and hence the following relation is obtained. (Source is moving with relativistic speed)

$$\Delta t_o = \gamma \Delta t_s + \frac{\Delta l}{c}$$

$$\Delta l = \frac{v\gamma \Delta t_s}{c}$$

$$\frac{\Delta t_o}{\Delta t_s} = \frac{1 + {}^{v}/_{c}}{\sqrt{1 - \left({}^{v^2}/_{c^2}\right)}}$$

On further simplification, we get,

$$\boxed{1 + z = \sqrt{\frac{1 + {}^{v}/_{c}}{1 - {}^{v}/_{c}}}}$$

The above expression is known as the **Kinematic Doppler Shift Expression**.

Case 4: Relativistic Case of Observer Moving

Refer to the same figure as in Case II. Due to relativistic effect, time shortening is observed and hence the following relation is obtained. (Observer is moving with relativistic speed)

$$\Delta t_o = \frac{\Delta t_s}{\gamma} + \frac{\Delta l}{c}$$

$$\Delta l = \frac{v \Delta t_o}{c}$$

$$\frac{\Delta t_o}{\Delta t_s} = \frac{\sqrt{1 - \left({}^{v^2}/_{c^2}\right)}}{1 - {}^{v}/_{c}}$$

On further simplification, we get –

$$\boxed{1 + z = \sqrt{\frac{1 + {}^{v}/_{c}}{1 - {}^{v}/_{c}}}}$$

The above expression is the same as what we got for Case III.

Points to Remember

- Recessional velocity and redshift of a star are related quantities.

- In a non-relativistic case, we cannot determine whether the source is moving or stationary.
- In a relativistic case, there is no difference in the redshift-recessional velocity relationship for source or observer moving.
- Moving clocks move slower, is a direct result of relativity theory.

4. Cosmology – Redshift vs. Kinematic Doppler Shift

A galaxy which is at redshift **z=10**, corresponds to **v≈80%** of **c**. The mass of the Milky Way is around **1011**$M_\odot$, if we consider the dark matter, it is **1012**$M_\odot$. Our milky way is thus massive. If it moves at 80% of **c**, it does not fit in the general concept of how objects move.

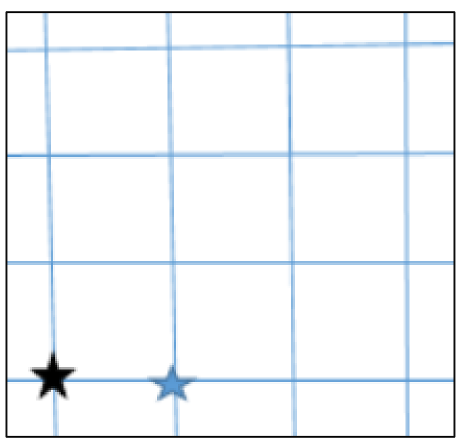
We know,

$$\frac{v_r}{c} = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}}$$

For small values of z,

$$z = \frac{v_r}{c} = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}}$$

In the following graph, the class between flux and wavelength, there are emission lines on top of the continuum. From the **H-α** line information, we get to conclude that roughly **z=7**. This implies galaxy is moving at 70% of **c**. We are observing a shift and interpreting it as velocity. We should get rid of this notion and look at **z** in a different way. Imagine space as a 2D grid representing the universe as shown below.

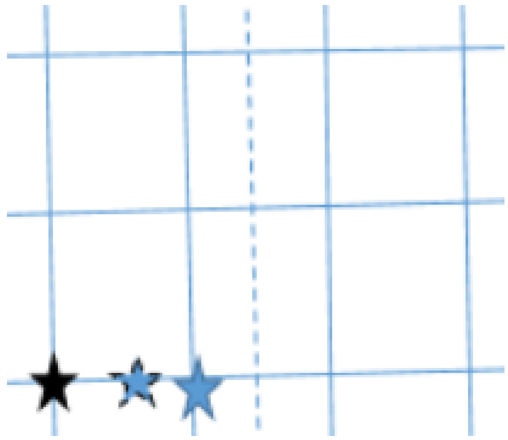


Consider the black star to be our own milky way and the blue star to be some other galaxy. When we record light from this galaxy, we see the spectrum and find out its redshift i.e., the galaxy is moving away. When the photon was emitted, it had relative velocity.

- What if the space was expanding?

- It is an instantaneous redshift of photon. Cumulative redshifts along the space between two galaxies will tend to a large redshift. The wavelength will change finally. It is the expansion of the space rather than the kinematic movement of the galaxies.

The following image shows if mutual gravity overflows expansion then this is not participating in the Hubble's law.



In the Kinematic Doppler Shift, the redshift is induced in a photon at the time of emission. In a Cosmological Redshift, in every step, it is getting cumulatively redshifted. In a gravitational potential, a photon will get blue shifted. As it crawls out of gravitational potential, it gets redshifted.

As per a Special Theory of Relativity, two objects passing by each other cannot have a relative velocity greater than the speed of light. The velocity we speak about is of the expansion of the universe. For large values of **z**, the redshift is cosmological and not a valid measure of the actual recessional velocity of the object with respect to us.

The Cosmological Principle

It stems from the **Copernicus Notion** of the universe. As per this notion, the universe is homogeneous and isotropic. There is no preferred direction and location in the universe.

- Homogeneity means no matter which part of the universe you reside in, you will see universe is the same in all parts. Isotropic nature means no matter which direction you look, you are going to see the same structure.

- A fitting example of homogeneity is a Paddy field. It looks homogeneous from all parts, but when wind flows, there are variations in its orientation, thus it is not isotropic. Consider a mountain on a flat land and an observer is standing on the mountain top. He will see the isotropic nature of the flat land, but it is not homogeneous. If in a homogeneous universe, it is isotropic at a point, it is isotropic everywhere.

- There have been large scale surveys to map the universe. **Sloan Digital Sky Survey** is one such survey, which did not focus much on the declination, but on the right ascension. The lookback time is around 2 billion years. Every pixel corresponds to the location of a galaxy and the color corresponds to the morphological structure. The green color represented blue spiral galaxy while the red false color indicated massive galaxies.

- Galaxies are there in a filamentary structure in a cosmological web and there are voids in between the galaxies.
- $\delta M/M \cong 1$ i.e., fluctuation of mass distribution is 1 M is the mass of the matter present within a given cube. In this case, take the volume 50 Mpc cube.
- For a cube side of 1000 Mpc, $\delta M/M \cong 10^{-4}$.
- One way to quantify homogeneity is to take mass fluctuations. Mass fluctuations will be higher at lower scales.
- For quantifying the isotropic nature, consider cosmic microwave background radiation. Universe is nearly isotropic at large angular scales.

Points to Remember

- Two objects passing by each other cannot have a relative velocity greater than the speed of light.
- Cosmological Principle states that the universe is homogeneous and isotropic.
- This homogeneity exists at a very large angular scale and not on smaller scales.
- SDSS (Sloan Digital Sky Survey) is an effort to map the night sky, verifying the Cosmological Principle.

5. Cosmology – Cosmological Metric & Expansion

As per the law of conservation of energy and the law of conservation of mass, the total amount of energy including the mass ($E=mc^2$) remains unchanged throughout every step in any process in the universe. The expansion of the universe itself consumes energy which maybe from the stretching of wavelength of photons (Cosmological Redshift), Dark Energy Interactions, etc.

To speed up the survey of more than 26,000 galaxies, **Stephen A. Shectman** designed an instrument capable of measuring 112 galaxies simultaneously. In a metal plate, holes that corresponded to the positions of the galaxies in the sky were drilled. Fiber-optic cables carried the light from each galaxy down to a separate channel on a spectrograph at the 2.5-meter du Pont telescope at the **Carnegie Observatories** on Cerro Las Campanas in Chile.

For maximum efficiency, a specialized technique known as the **Drift-Scan Photometry** was used, in which the telescope was pointed at the beginning of a survey field and then automated drive was turned off. The telescope stood still as the sky drifted past. Computers read information from the **CCD Detector** at the same rate as the rotation of the earth, producing one long, continuous image at a constant celestial latitude. Completing the photometry took a total of 450 hours.

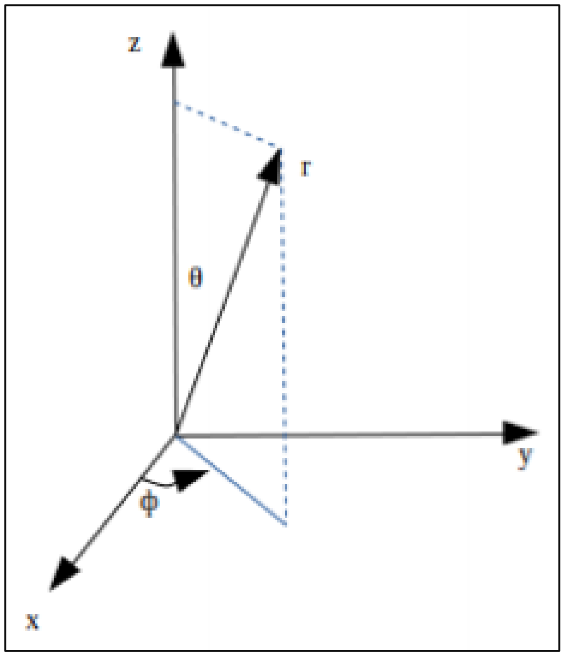
Different forms of noise exist and their mathematical modelling is different depending upon its properties. Various physical processes evolve the power spectrum of the universe on a large scale. The initial power spectrum imparted due to the quantum fluctuations follows a negative third power of frequency which is a form of **Pink Noise Spectrum** in three dimensions.

The Metric

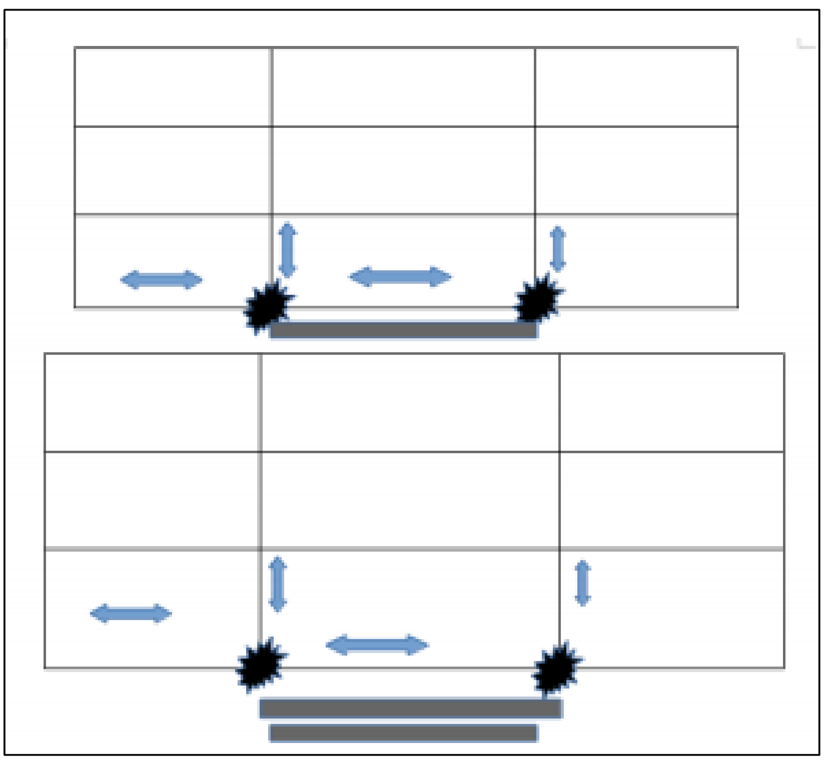
In cosmology, one must first have a definition of space. A metric is a mathematical expression describing points in space. The observation of the sky is done in a spherical geometry; hence a spherical coordinate system shall be used. The distance between two closely spaced points is given by –

$$ds^2 = dr^2 + r^2\theta^2 + r^2 sin^2\theta d\phi^2$$

The following image shows Geometry in the 3-dimensional non-expanding Euclidean space.



This geometry is still in the 3-dimensional non-expanding Euclidean space. Hence, the reference grid defining the frame itself would be expanding. The following image depicts the increased metric.



A scale factor is put into the equation of the non-expanding space, called the 'scale factor' which incorporates expansion of the universe with respect to time.

$$ds^2 = a^2(t)[dr^2 + r^2\theta^2 + r^2 sin^2\theta d\phi^2]$$

where **a(t)** is the scale factor, sometimes written as **R(t)**. Whereas, **a(t) > 1** means magnification of the metric, while **a(t) < 1** means shrinking of the metric and **a(t) =1** means constant metric. As a convention, **$a(t_0)=1$**.

Comoving Coordinate System

In a **Comoving Coordinate System**, the measuring scale expands along with the frame (expanding universe).

Here, the $[dr^2 + r^2 \theta^2 + r^2 \sin^2\theta \, d\varphi^2]$ is the Comoving Distance, and the **ds^2** is the Proper distance.

The proper distance will correspond to the actual distance as measured of a distant galaxy from earth at the time of observation, a.k.a. instantaneous distance of objects.

This is because the distance travelled by a photon when it reaches the observer from a distant source will be the one received at **$t=t_0$** of the observer, which would mean that the instantaneous observed distance will be the proper distance, and one can predict future distances using the rate factor and the initial measured length as a reference.

The concept of Comoving and proper distance is important in measuring the actual value of the number density of galaxies in the given volume of the observed space. One must use the Comoving distance to calculate the density at the time of their formation when the observed photon was emitted. That can be obtained once the rate of expansion of the universe can be estimated.

To estimate the rate of expansion, one can observe the change in the distance of an observed distant galaxy over a long period of time.

Points to Remember

- A metric is a mathematical expression describing the points in space.
- The scale factor determines whether the universe is contracting or expanding.
- In a comoving coordinate system, the measuring scale expands along with the frame (expanding universe).
- Proper distance is the instantaneous distance of objects.
- Comoving distance is the actual distance of objects.