Network Theory

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About the Tutorial

This tutorial is meant to provide the readers the know-how to analyze and solve any electric circuit or network. After completing this tutorial, you will understand the laws and methods that can be applied to specific electric circuits and networks.

Audience

This tutorial is meant for all the readers who are aspiring to learn the concepts of Network Theory. In some universities, this subject is also called as "Network Analysis & Circuit Theory."

Prerequisites

There are no major prerequisites to understand the concepts discussed in this tutorial. Once you are through with the first few chapters, you will be quite at ease with the methods and concepts of DC circuits and AC circuits, discussed in later chapters.

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1. Network Theory – Overview

Network theory is the study of solving the problems of electric circuits or electric networks. In this introductory chapter, let us first discuss the basic terminology of electric circuits and the types of network elements.

Basic Terminology

In Network Theory, we will frequently come across the following terms:

- Electric Circuit
- Electric Network
- Current
- Voltage
- Power

So, it is imperative that we gather some basic knowledge on these terms before proceeding further. Let's start with Electric Circuit.

Electric Circuit

An electric circuit contains a closed path for providing a flow of electrons from a voltage source or current source. The elements present in an electric circuit will be in **series connection**, **parallel connection**, or in any combination of series and parallel connections.

Electric Network

An electric network need not contain a closed path for providing a flow of electrons from a voltage source or current source. Hence, we can conclude that "all electric circuits are electric networks" but the converse need not be true.

Current

The current "**I**" flowing through a conductor is nothing but the time rate of flow of charge. Mathematically, it can be written as

$$I = \frac{dQ}{dt}$$

Where,

- **Q** is the charge and its unit is Coloumb.
- **t** is the time and its unit is second.

As an analogy, electric current can be thought of as the flow of water through a pipe. Current is measured in terms of **Ampere**.

In general, **Electron current** flows from negative terminal of source to positive terminal, whereas, **Conventional current** flows from positive terminal of source to negative terminal.

Electron current is obtained due to the movement of free electrons, whereas, **Conventional current** is obtained due to the movement of free positive charges. Both of these are called as **electric current**.

Voltage

The voltage "V" is nothing but an electromotive force that causes the charge (electrons) to flow. Mathematically, it can be written as

$$V = \frac{dW}{dQ}$$

Where,

- **W** is the potential energy and its unit is Joule.
- **Q** is the charge and its unit is Coloumb.

As an analogy, Voltage can be thought of as the pressure of water that causes the water to flow through a pipe. It is measured in terms of **Volt**.

Power

The power "P" is nothing but the time rate of flow of electrical energy. Mathematically, it can be written as

$$P = \frac{dW}{dt}$$

Where,

- **W** is the electrical energy and it is measured in terms of **Joule**.
- **t** is the time and it is measured in seconds.

We can re-write the above equation as

$$P = \frac{dW}{dt} = \frac{dW}{dQ} \times \frac{dQ}{dt} = VI$$

Therefore, **power** is nothing but the **product** of voltage **V** and current **I**. Its unit is **Watt**.

Types of Network Elements

We can classify the Network elements into various types based on some parameters. Following are the types of Network elements:

- Active Elements and Passive Elements
- Linear Elements and Non-linear Elements
- Bilateral Elements and Unilateral Elements

Active Elements and Passive Elements

We can classify the Network elements into either **active** or **passive** based on the ability of delivering power.

- **Active Elements** deliver power to other elements, which are present in an electric circuit. Sometimes, they may absorb the power like passive elements. That means active elements have the capability of both delivering and absorbing power. **Examples:** Voltage sources and current sources.

- **Passive Elements** can't deliver power (energy) to other elements, however they can absorb power. That means these elements either dissipate power in the form of heat or store energy in the form of either magnetic field or electric field. **Examples:** Resistors, Inductors, and capacitors.

Linear Elements and Non-Linear Elements

We can classify the network elements as **linear** or **non-linear** based on their characteristic to obey the property of linearity.

- **Linear Elements** are the elements that show a linear relationship between voltage and current. **Examples:** Resistors, Inductors, and capacitors.

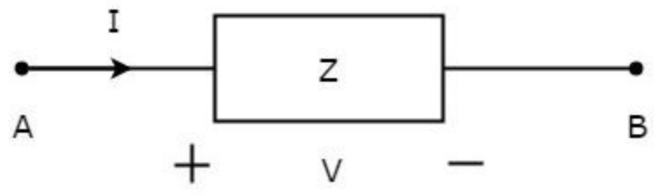
- **Non-Linear Elements** are those that do not show a linear relation between voltage and current. **Examples:** Voltage sources and current sources.

Bilateral Elements and Unilateral Elements

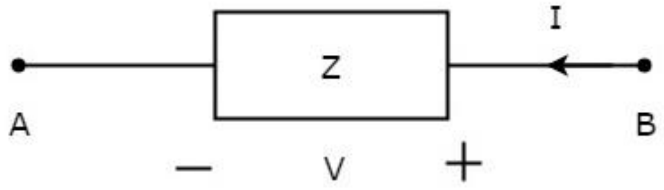
Network elements can also be classified as either **bilateral** or **unilateral** based on the direction of current flows through the network elements.

Bilateral Elements are the elements that allow the current in both directions and offer the same impedance in either direction of current flow. **Examples:** Resistors, Inductors and capacitors.

The concept of Bilateral elements is illustrated in the following figures.



In the above figure, the current (I) is flowing from terminals A to B through a passive element having impedance of $Z\,\Omega$. It is the ratio of voltage (V) across that element between terminals A & B and current (I).



In the above figure, the current (I) is flowing from terminals B to A through a passive element having impedance of $Z\,\Omega$. That means the current (–I) is flowing from terminals A to B. In this case too, we will get the same impedance value, since both the current and voltage having negative signs with respect to terminals A & B.

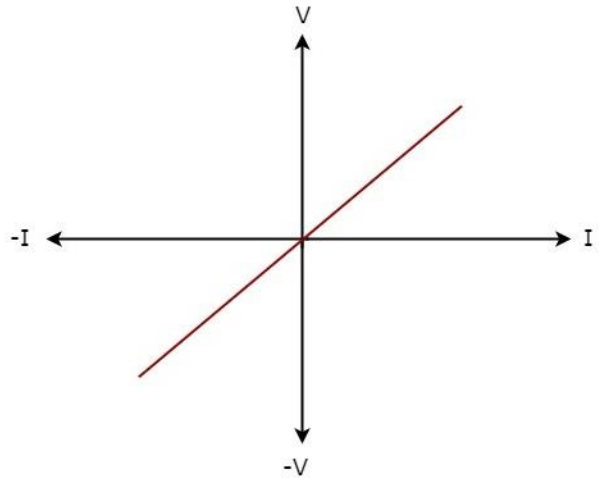
Unilateral Elements are those that allow the current in only one direction. Hence, they offer different impedances in both directions.

2. Network Theory – Example Problems

We discussed the types of network elements in the previous chapter. Now, let us identify the **nature of network elements** from the V-I characteristics given in the following examples.

Example 1

The **V-I characteristics** of a network element is shown below.



Step 1: Verifying the network element as **linear** or **non-linear**.

From the above figure, the V-I characteristics of a network element is a straight line passing through the origin. Hence, it is a **Linear element**.

Step 2: Verifying the network element as **active** or **passive**.

The given V-I characteristics of a network element lies in the first and third quadrants.

- In the **first quadrant**, the values of both voltage (V) and current (I) are positive. So, the ratios of voltage (V) and current (I) gives positive impedance values.

- Similarly, in the **third quadrant**, the values of both voltage (V) and current (I) have negative values. So, the ratios of voltage (V) and current (I) produce positive impedance values.

Since, the given V-I characteristics offer positive impedance values, the network element is a **Passive element**.

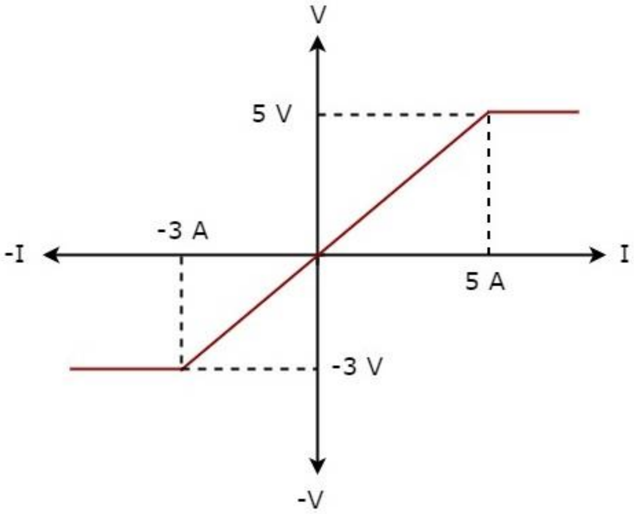
Step 3: Verifying the network element as **bilateral** or **unilateral**.

For every point (I, V) on the characteristics, there exists a corresponding point (-I, -V) on the given characteristics. Hence, the network element is a **Bilateral element**.

Therefore, the given V-I characteristics show that the network element is a **Linear**, **Passive**, and **Bilateral element**.

Example 2

The **V-I characteristics** of a network element is shown below.



Step 1: Verifying the network element as **linear** or **non-linear**.

From the above figure, the V-I characteristics of a network element is a straight line only between the points (-3A, -3V) and (5A, 5V). Beyond these points, the V-I characteristics are not following the linear relation. Hence, it is a **Non-linear element**.

Step 2: Verifying the network element as **active** or **passive**.

The given V-I characteristics of a network element lies in the first and third quadrants. In these two quadrants, the ratios of voltage (V) and current (I) produce positive impedance values. Hence, the network element is a **Passive element**.

Step 3: Verifying the network element as **bilateral or unilateral**.

Consider the point (5A, 5V) on the characteristics. The corresponding point (-5A, -3V) exists on the given characteristics instead of (-5A, -5V). Hence, the network element is a **Unilateral element**.

Therefore, the given V-I characteristics show that the network element is a **Non-linear**, **Passive**, and **Unilateral element**.

3. Network Theory – Active Elements

Active Elements are the network elements that deliver power to other elements present in an electric circuit. So, active elements are also called as sources of voltage or current type. We can classify these sources into the following two categories:

- Independent Sources
- Dependent Sources

Independent Sources

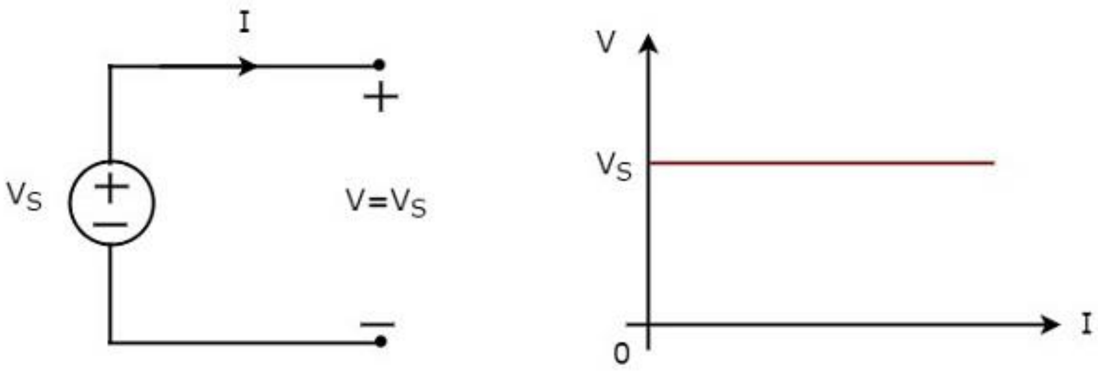
As the name suggests, independent sources produce fixed values of voltage or current and these are not dependent on any other parameter. Independent sources can be further divided into the following two categories:

- Independent Voltage Sources
- Independent Current Sources

Independent Voltage Sources

An independent voltage source produces a constant voltage across its two terminals. This voltage is independent of the amount of current that is flowing through the two terminals of voltage source.

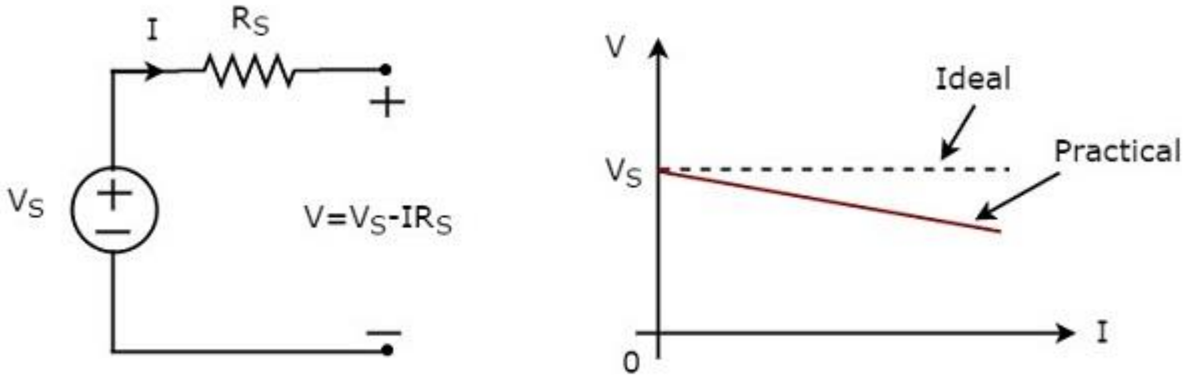
Independent **ideal voltage source** and its V-I characteristics are shown in the following figure.



The **V-I characteristics** of an independent ideal voltage source is a constant line, which is always equal to the source voltage (V_S) irrespective of the current value (I). So, the internal resistance of an independent ideal voltage source is zero Ohms.

Hence, the independent ideal voltage sources **do not exist practically**, because there will be some internal resistance.

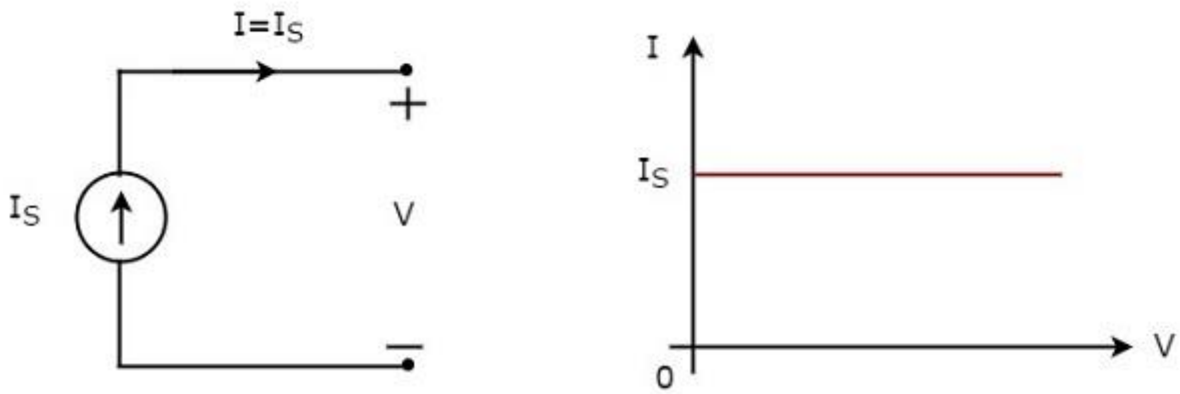
Independent **practical voltage source** and its V-I characteristics are shown in the following figure.



There is a deviation in the V-I characteristics of an independent practical voltage source from the V-I characteristics of an independent ideal voltage source. This is due to the voltage drop across the internal resistance (R_S) of an independent practical voltage source.

Independent Current Sources

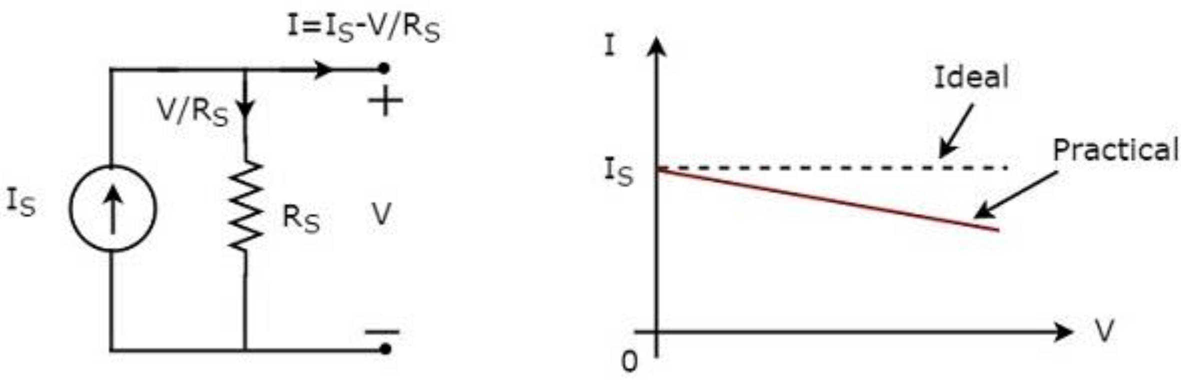
An independent current source produces a constant current. This current is independent of the voltage across its two terminals. Independent **ideal current source** and its V-I characteristics are shown in the following figure.



The **V-I characteristics** of an independent ideal current source is a constant line, which is always equal to the source current (I_S) irrespective of the voltage value (V). So, the internal resistance of an independent ideal current source is infinite ohms.

Hence, the independent ideal current sources **do not exist practically**, because there will be some internal resistance.

Independent **practical current source** and its V-I characteristics are shown in the following figure.



There is a deviation in the V-I characteristics of an independent practical current source from the V-I characteristics of an independent ideal current source. This is due to the amount of current flows through the internal shunt resistance (R_S) of an independent practical current source.

Dependent Sources

As the name suggests, dependent sources produce the amount of voltage or current that is dependent on some other voltage or current. Dependent sources are also called as **controlled sources**. Dependent sources can be further divided into the following two categories:

- Dependent Voltage Sources
- Dependent Current Sources

Dependent Voltage Sources

A dependent voltage source produces a voltage across its two terminals. The amount of this voltage is dependent on some other voltage or current. Hence, dependent voltage sources can be further classified into the following two categories:

- Voltage Dependent Voltage Source (VDVS)
- Current Dependent Voltage Source (CDVS)

Dependent voltage sources are represented with the signs '+' and '-' inside a diamond shape. The magnitude of the voltage source can be represented outside the diamond shape.

Dependent Current Sources

A dependent current source produces a current. The amount of this current is dependent on some other voltage or current. Hence, dependent current sources can be further classified into the following two categories:

- Voltage Dependent Current Source (VDCS)
- Current Dependent Current Source (CDCS)

Dependent current sources are represented with an arrow inside a diamond shape. The magnitude of the current source can be represented outside the diamond shape.

We can observe these dependent or controlled sources in equivalent models of transistors.

Source Transformation Technique

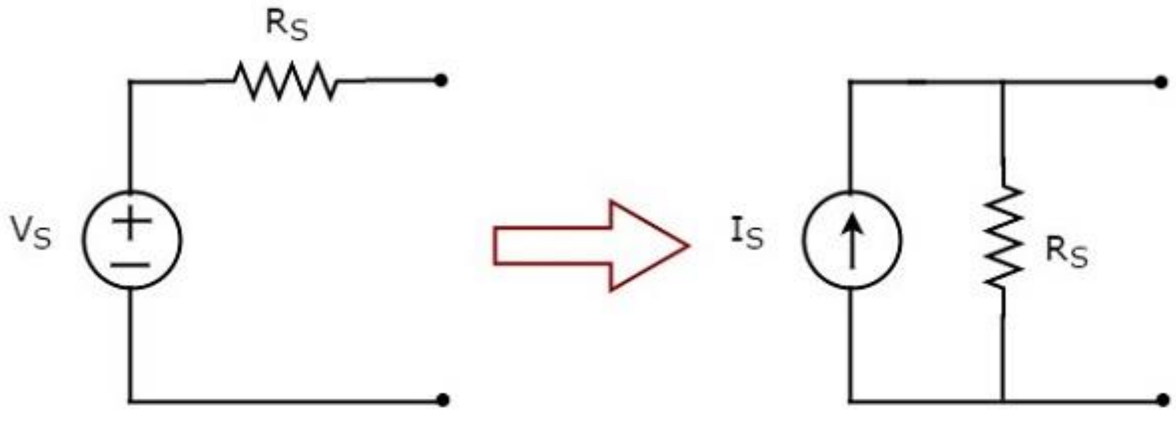
We know that there are two practical sources, namely, **voltage source** and **current source**. We can transform (convert) one source into the other based on the requirement, while solving network problems.

The technique of transforming one source into the other is called as **source transformation technique**. Following are the two possible source transformations:

- Practical voltage source into a practical current source
- Practical current source into a practical voltage source

Practical voltage source into a practical current source

The transformation of practical voltage source into a practical current source is shown in the following figure.



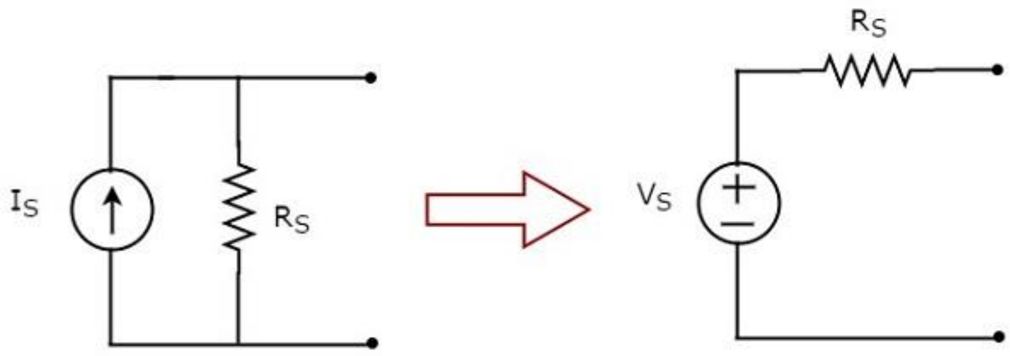
Practical voltage source consists of a voltage source (V_S) in series with a resistor (R_S). This can be converted into a practical current source as shown in the figure. It consists of a current source (I_S) in parallel with a resistor (R_S).

The value of I_S will be equal to the ratio of V_S and R_S. Mathematically, it can be represented as

$$I_S = \frac{V_S}{R_S}$$

Practical current source into a practical voltage source

The transformation of practical current source into a practical voltage source is shown in the following figure.



Practical current source consists of a current source (I_S) in parallel with a resistor (R_S). This can be converted into a practical voltage source as shown in the figure. It consists of a voltage source (V_S) in series with a resistor (R_S).

The value of V_S will be equal to the product of I_S and R_S. Mathematically, it can be represented as

$$V_S = I_S R_S$$

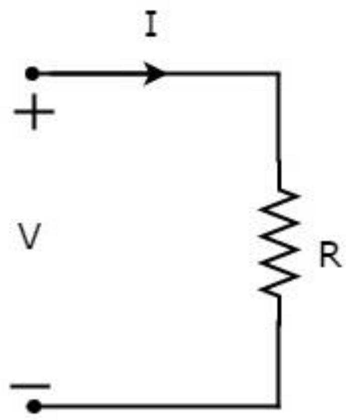
4. Network Theory – Passive Elements

In this chapter, we will discuss in detail about the passive elements such as Resistor, Inductor, and Capacitor. Let us start with Resistors.

Resistor

The main functionality of Resistor is either opposes or restricts the flow of electric current. Hence, the resistors are used in order to limit the amount of current flow and / or dividing (sharing) voltage.

Let the current flowing through the resistor is I amperes and the voltage across it is V volts. The **symbol** of resistor along with current, I and voltage, V are shown in the following figure.



According to **Ohm's law**, the voltage across resistor is the product of current flowing through it and the resistance of that resistor. **Mathematically**, it can be represented as

$$V = IR$$ **Equation 1**

$$=> I = \frac{V}{R}$$ **Equation 2**

Where, **R** is the resistance of a resistor.

From Equation 2, we can conclude that the current flowing through the resistor is directly proportional to the applied voltage across resistor and inversely proportional to the resistance of resistor.

Power in an electric circuit element can be represented as

$$P = VI$$ **Equation 3**

Substitute, Equation 1 in Equation 3.

$$P = (IR)I$$

$$=> P = I^2\, R$$ **Equation 4**

Substitute, Equation 2 in Equation 3.

$$P = V\left(\frac{V}{R}\right)$$

$$=> P = \frac{V^2}{R}$$ **Equation 5**

So, we can calculate the amount of power dissipated in the resistor by using one of the formulae mentioned in Equations 3 to 5.

Inductor

In general, inductors will have number of turns. Hence, they produce magnetic flux when current flows through it. So, the amount of total magnetic flux produced by an inductor depends on the current, I flowing through it and they have linear relationship.

Mathematically, it can be written as

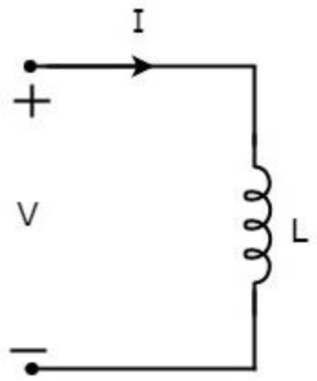
$$\Psi \; \alpha \; I$$

$$=> \Psi = LI$$

Where,

- $\boldsymbol{\Psi}$ is the total magnetic flux
- $\boldsymbol{L}$ is the inductance of an inductor

Let the current flowing through the inductor is I amperes and the voltage across it is V volts. The **symbol** of inductor along with current I and voltage V are shown in the following figure.



According to **Faraday's law**, the voltage across the inductor can be written as

$$V = \frac{d\Psi}{dt}$$

Substitute $\Psi = L\,I$ in the above equation.

$$V = \frac{d(LI)}{dt}$$

$$=> V = L\frac{dI}{dt}$$

$$=> I = \frac{1}{L}\int V\,dt$$

From the above equations, we can conclude that there exists a **linear relationship** between voltage across inductor and current flowing through it.

We know that **power** in an electric circuit element can be represented as

$$P = VI$$

Substitute $V = L\frac{dI}{dt}$ in the above equation.

$$P = \left(L\frac{dI}{dt}\right)I$$

$$=> P = LI\frac{dI}{dt}$$

By integrating the above equation, we will get the **energy** stored in an inductor as

$$W = \frac{1}{2}LI^2$$

So, the inductor stores the energy in the form of magnetic field.

Capacitor

In general, a capacitor has two conducting plates, separated by a dielectric medium. If positive voltage is applied across the capacitor, then it stores positive charge. Similarly, if negative voltage is applied across the capacitor, then it stores negative charge.

So, the amount of charge stored in the capacitor depends on the applied voltage **V** across it and they have linear relationship. Mathematically, it can be written as

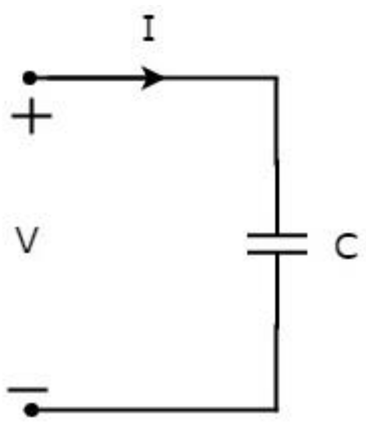
$$Q \; \alpha \; V$$

$$=> Q = CV$$

Where,

- $\boldsymbol{Q}$ is the charge stored in the capacitor.
- $\boldsymbol{C}$ is the capacitance of a capacitor.

Let the current flowing through the capacitor is I amperes and the voltage across it is V volts. The symbol of capacitor along with current I and voltage V are shown in the following figure.



We know that the **current** is nothing but the **time rate of flow of charge**. Mathematically, it can be represented as

$$I = \frac{dQ}{dt}$$

Substitute $Q = CV$ in the above equation.

$$I = \frac{d(CV)}{dt}$$

$$=> I = C\frac{dV}{dt}$$

$$=> V = \frac{1}{C}\int I\,dt$$

From the above equations, we can conclude that there exists a **linear relationship** between voltage across capacitor and current flowing through it.

We know that **power** in an electric circuit element can be represented as

$$P = VI$$

Substitute $I = C\frac{dV}{dt}$ in the above equation.

$$P = V\left(C\frac{dV}{dt}\right)$$

$$=> P = CV\frac{dV}{dt}$$

By integrating the above equation, we will get the **energy** stored in the capacitor as

$$W = \frac{1}{2}CV^2$$

So, the capacitor stores the energy in the form of electric field.

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